**Homework 8 - Maximum Likelihood**

**Due November 14 at 9:00am**

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**Submit HW as a link to a GitHub repository containing your code and a pdf of this worksheet**

**Background**

We’re going to implement the three ways of fitting a linear regression model that were mentioned in lecture: (1) the analytical solution using the normal equation, (2) a numerical optimization approach based on minimizing the sum of squared errors, and (3) a numerical optimization approach based on maximizing the likelihood.

# Q1: Simulate some data for a linear regression using the simple linear regression model: yi = beta\_0 + beta\_1\*xi + error

Where error~N(0,sigma).

Make sigma large enough that it resembles “typical” ecological data when you plot it, but not so large that it completely obscures the relationship between x and y.

Record your true beta\_0, beta\_1, and sigma here:

**Sigma = 2**

**Beta\_0 = 5**

**Beta\_1 = 0.8**

Fit a linear regression model using lm() and do your typical checks of model assumptions that we went over in class.

Chart, scatter chart

Description automatically generated

Looks good! No obvious trends or major outliers detected.

Paste your estimated model coefficients and their 95% confidence intervals below:

**Estimate Std. Error t value Pr(>|t|)**

**(Intercept) 3.9052 0.9516 4.104 8.41e-05 \*\*\***

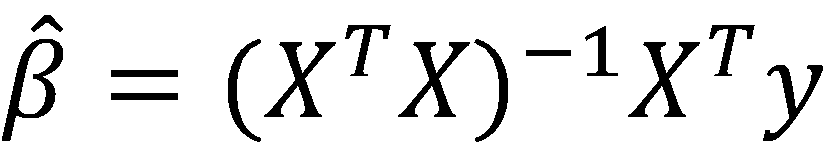
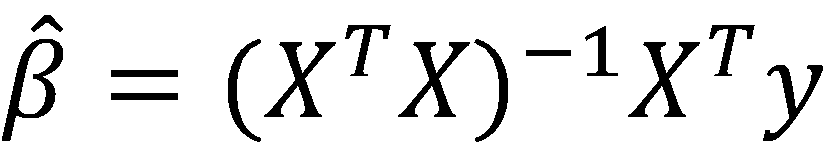
**x 0.8702 0.0618 14.083 < 2e-16 \*\*\***

Are the coefficient estimates close to the true values? Do the 95% confidence intervals cover the true values?

**Yes! Beta\_1 is estimated to be 0.87, while the true value is 0.8. Beta\_0 is estimated at 3.9, while the true value is 5. Beta\_1 is within the confidence interval but Beta\_0 is not**

Relevant functions: rnorm()

# Q2: Analyze the data generated in Q1 using the normal equation:



Paste your estimated model coefficients below.

**[,1]**

**[1,] 1.09556**

Are the coefficient estimates close to the true values?

**The estimate for the slope was a little higher than the true value by about 0.2.**

# Bonus Q1: Can you find an analytical solution for the se and 95% CI for the model coefficients? (note: this is not in the lectures or reading, you'll have to do some searching). Write the equations and resulting answers below. Do the 95% confidence intervals cover the true values?

Relevant functions: solve(), t()

# Q3: Analyze the data generated in Q1 using a grid search to ***minimize the sum of squared errors*** (no need to iterate more than twice):

Paste your estimated model coefficients below.

**> min\_coefs**

**[1] 4.6 0.8**

**Intercept = 4.6**

**Slope = 0.8**

# Q4: Analyze the data generated in Q1 using a grid search to ***minimize the negative log likelihood*** (no need to iterate more than twice). Note, there is a third parameter that you will need to estimate here: sigma

Paste your estimated model coefficients below.

**> min\_nll\_coefs**

**[1] 4.600 0.801 0.081**

**Intercept = 4.6**

**Slope = 0.801**

**Sigma = 0.081**

# Q5: Analyze the data generated in Q1 ***using optim()*** to minimize the negative log likelihood. Note, there is a third parameter that you will need to estimate here: sigma

Paste your estimated model coefficients below.

Text

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Did the numerical optimization algorithm converge? How do you know?

It did, “0” indicates a successful optimization

Did the numerical optimization algorithm find a global solution? How do you know?

Chart, scatter chart

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Seems to be a global optimization, the regression seems to fit the entire dataset when plotted.

# Q6: Plot a likelihood profile for the slope parameter while estimating the conditional MLEs of the intercept and sigma for each plotted value of the slope parameter (see p. 173 of Hilborn and Mangel).

Chart

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Plot is of MLE of intercept and sigma, horizontal line is of the MLE minimum and the red dot is of the optim derived slope value (0.84) along the curve.

# Q7: Plot the joint likelihood surface for the intercept and slope parameters. Is there evidence of confounding between these two parameters (i.e., a ridge rather than a mountain top)?

Diagram

Description automatically generated

Seems to be a mountain top, no local maximums

# Q8: How different are the estimated coefficients from Q1, Q2, and Q5 and how do they compare to the true values?

**Sigma = 2**

**Beta\_0 = 5**

**Beta\_1 = 0.8**

Questions 1 and 2 were done at the same time and Q5 occurred with a different dataset, so a true comparison is difficult to make. But overall, it seemed that as the complexity of finding optimum values increased so did their similarity to their true values.

# Bonus Q2: Calculate the standard errors of the intercept, slope, and sigma using the Hessian matrix. Standard errors are the square roots of the diagonal of the inverse Hessian matrix. How do these standard errors compare to those from lm() in Q1?

# Bonus Q3: How does the computational speed compare between using lm(), the normal equation, and optim() to estimate the coefficients? Note, you can get the computation time placing the following code around your regression code:

# Record your system start time

start\_time <- proc.time()

#your code goes here

# Subtract start time from current system time

proc.time() – start\_time